ON INDEFINITE INTEGRAL

(MANUAL PARA CALCULAR ANTIDERIVADAS E INTEGRALES DEFINIDAS EN HOJA DE CÁLCULO)

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NOTA: Este libro está en tamaño carta

Se usó el software **WPS Office**

ANTIDERIVADAS

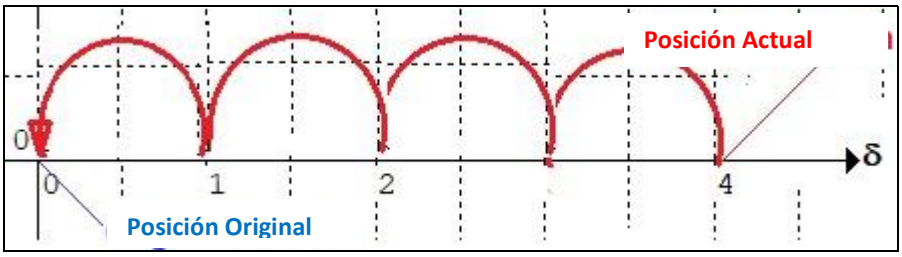
SOLUCIONANDO ANTIDERIVADAS CON HOJA DE CÁLCULO

Encuentra la Antiderivada de:

$$\frac{d^4}{dx^4} = 720x$$

Condición Inicial: (x,y,y',y'',y''')=(1,14; 36,126,360)

Sol…



Podemos ver que la función derivada está 4 unidades a la derecha de el Origen (***Función Original***), así que debemos movernos 4 unidades a la izquierda para llegar a la ***Función Original***.

Para hacer esto, usamos la siguiente definición:

$$\rightarrow \frac{d^{(\delta-m)}}{dx^{(\delta-m)}}y = \frac{d^{-m}}{dx^{-m}}f(x) + \sum_{k=1}^{m}\left[C_k.\frac{d^{-(\boldsymbol{m}-k)}}{dx^{-(\boldsymbol{m}-k)}}x^0\right] \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(\Omega)$$

Donde las ***derivadas con signo negativo*** representan ***integrales***.

NOTA: Del lado derecho de la definición (***Ω***), tanto la parte que **NO** está dentro de los corchetes [ ], así como la parte que **SÍ** está dentro de los corchetes [ ], se obtuvieron de la ***Definición (1.1)*** (página 12) del libro "***On Indefinite Integral***".

Aunque podemos simplificar la anterior definición y expresarla del modo siguiente:

$$\rightarrow \frac{d^{(\delta-m)}}{dx^{(\delta-m)}}y = \frac{\boldsymbol{d^{-m}}}{\boldsymbol{dx^{-m}}}\boldsymbol{f(x)} + \sum_{k=1}^{m}\left[C_k.\frac{x^{(\boldsymbol{m}-k)}}{(m-k)!}\right] \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(\Omega 1)$$

$$\underbrace{\int\int\int\cdots\int\int\int \boldsymbol{f(x)dx^m}}_{\boldsymbol{m-veces}}$$

La expresión de arriba en su forma de tabla queda mejor comprendida:

	$\lim_{n\to\infty}\left\{\frac{f\left[x-(m)\left(\frac{x}{n}\right)\right]}{0!}+(m).\frac{f\left[x-(m+1)\left(\frac{x}{n}\right)\right]}{1!}\right.$ $+\frac{(m)(m+1)}{2!}.f\left[x-(m+2)\left(\frac{x}{n}\right)\right]$ $+\frac{(m)(m+1)(m+2)}{3!}.f\left[x-(m+3)\left(\frac{x}{n}\right)\right]$ $+\frac{(m)(m+1)(m+2)(m+3)}{4!}.f\left[x-(m+4)\left(\frac{x}{n}\right)\right]$ $+\frac{(m)(m+1)(m+2)(m+3)(m+4)}{5!}.f[x$ $\left.-(m+5)\left(\frac{x}{n}\right)\right]+\cdots\}.\left(\frac{x}{n}\right)^{m}$	$\cdots(\boldsymbol{\Omega 1})$
(+)	$\sum_{k=1}^{m}\left[C_k.\frac{x^{(\boldsymbol{m}-k)}}{(m-k)!}\right]$	
(=)	$\frac{\boldsymbol{d}^{(\boldsymbol{\delta}-\boldsymbol{m})}}{\boldsymbol{dx}^{(\boldsymbol{\delta}-\boldsymbol{m})}}\boldsymbol{y}$	

La expresión contenida en la tabla nos permitirá regresar de la ***función derivada cuatro veces*** a la ***función original***. Para hacer esto, debemos ir de uno en un pasos a la vez, hasta llegar a la ***función original***. De un modo más general, la **Definición (*Ω1*)** nos permite encontrar la Función Original aún cuando ***δ*** esté mucho más a la derecha del 4.

Por tanto, (para este caso):

Posición: Actual ***δ=4***

I) PRIMER PASO A LA IZQUIERDA

Vamos a dar el ***Primer paso a la izquierda***, por tanto ***m=1.***

	$\lim_{n\to\infty}\left\{\frac{f\left[x-(1)\left(\frac{x}{n}\right)\right]}{0!}+(1).\frac{f\left[x-(1+1)\left(\frac{x}{n}\right)\right]}{1!}+\frac{(1)(1+1)}{2!}.f\left[x-(1+2)\left(\frac{x}{n}\right)\right]\right.$ $+\frac{(1)(1+1)(1+2)}{3!}.f\left[x-(1+3)\left(\frac{x}{n}\right)\right]$ $+\frac{(1)(1+1)(1+2)(1+3)}{4!}.f\left[x-(1+4)\left(\frac{x}{n}\right)\right]$ $+\frac{(1)(1+1)(1+2)(1+3)(1+4)}{5!}.f\left[x-(1+5)\left(\frac{x}{n}\right)\right]$ $\left.+\cdots\right\}.\left(\frac{x}{n}\right)^{1}$	$\cdots(\boldsymbol{\Omega 1})$
(+)	$\sum_{k=1}^{1}\left[C_k.\frac{x^{(1-k)}}{(1-k)!}\right]$	
(=)	$\boldsymbol{\frac{d^{(4-1)}}{dx^{(4-1)}}y}$	

	$\lim_{n\to\infty}\left\{f\left(x-\frac{x}{n}\right)+f\left[x-2\left(\frac{x}{n}\right)\right]+f\left[x-3\left(\frac{x}{n}\right)\right]+f\left[x-4\left(\frac{x}{n}\right)\right]+f\left[x-5\left(\frac{x}{n}\right)\right]+f\left[x-6\left(\frac{x}{n}\right)\right]\right.$ $\left.+\cdots\right\}.\left(\frac{x}{n}\right)$
(+)	$C_1.\frac{x^{(1-1)}}{(1-1)!}$
(=)	$\boldsymbol{\frac{d^3}{dx^3}y}$

	$\lim_{n\to\infty}\left\{\sum_{k=1}^{n}f\left[x-k\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)$
(+)	$C_1.\frac{x^0}{(0)!}$
(=)	$\boldsymbol{\frac{d^3}{dx^3}y}$

	$\lim_{n\to\infty}\left\{\sum_{k=1}^{n}f\left[x-k\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)$
(+)	C_1
(=)	$\boldsymbol{\frac{d^3}{dx^3}y}$

Por tanto:

$$\frac{d^3}{dx^3}y = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n} f\left[x-(k).\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)\right\} + C_1$$

$$\rightarrow \frac{d^3}{dx^3}y = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n} f\left[x-(k).\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)\right\} + C_1 \cdots\cdots(m1A)$$

La expresión en ROJO representa la *Primera Integral Indefinida*

Sin embargo, dado que la Hoja de Cálculo será usada para encontrar numéricamente la ***Función Original***, la última expresión de arriba **[*m1A*]** se convierte en lo siguiente (al elegir un número muy grande ***N*** que tienda al infinito):

$$f'''(x_0) \cong \left\{\left\{\sum_{k=1}^{N} f\left[x_0-\left(\frac{x_0}{N}\right).(k)\right]\right\}.\left(\frac{x_0}{N}\right)\right\} + C_1 \cdots\cdots(m1)$$

Resolviendo para C_1, tenemos:

$$\rightarrow C_1 \cong f'''(x_0) - \left\{\left\{\sum_{k=1}^{N} f\left[x_0-\left(\frac{x_0}{N}\right).(k)\right]\right\}.\left(\frac{x_0}{N}\right)\right\} \cdots\cdots(m1)$$

La función derivada es:

$$\rightarrow \frac{d^4}{dx^4} = 720x$$

Condición Inicial (x,y,y',y'',y''')=(1,14; 36,126,360)

Infinito Elegido Numéricamente: N=100,000

Por tanto, ***la ecuación (m1)*** se convierte en:

$$\rightarrow C_1 \cong f'''(x_0) - \left\{\left\{\sum_{k=1}^{N} f\left[x_0-\left(\frac{x_0}{N}\right).(k)\right]\right\}.\left(\frac{x_0}{N}\right)\right\} \cdots\cdots(m1)$$

$$C_1 \cong 360 - \left\{\left\{\sum_{k=1}^{100,000} 720\left[1-\left(\frac{1}{100,000}\right).(k)\right]\right\}.\left(\frac{1}{100,000}\right)\right\} \cdots\cdots(M1)$$

CALCULANDO M=1 EN HOJA DE CÁLCULO

Abrimos una Hoja de Cálculo y la nombramos ***M=1***

La ecuación ***(M1)*** es:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100{,}000} \left\{ 720 \left[1 - \left(\frac{1}{100{,}000} \right) . (k) \right] . \left(\frac{1}{100{,}000} \right) \right\} \right\} \cdots\cdots (M1)$$

1) Escribimos esta ecuación en la **celda A1**:

2) En la **celda A10**, comenzamos a escribir lo siguiente:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100{,}000} \left\{ 720 \left[1 - \left(\frac{1}{100{,}000} \right) . (k) \right] . \left(\frac{1}{100{,}000} \right) \right\} \right\} \cdots\cdots (M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	=(B10/B11)		
13	f'''(x_0)=	360		
14				
15				
16				
17				
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19				
20				
21				

3) En la **celda A20**, comenzamos a poner lo siguiente:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21				
22				
23				

De acuerdo a:

$$C_1 \cong f'''(x_0) - \left\{ \sum_{k=1}^{N} \left\{ f \left[x_0 - \left(\frac{x_0}{N} \right) . (k) \right] . \left(\frac{x_0}{N} \right) \right\} \right\} \text{ ……} (m1)$$

$\left[x_0 - \left(\frac{x_0}{N} \right) . (k) \right]$ Representa la ***x modificada*** en **(M1)**.

$f \left[x_0 - \left(\frac{x_0}{N} \right) . (k) \right]$ Representa esa ***x modificada*** sustituida en la función ***f(x)***

4) De acuerdo a la ecuación **(M1)**, el rango de ***k*** va de ***k=1*** a ***k= 100,000*** ya que la ***N*** escogida para este caso es igual a ***N=100,000*** (un número muy grande para ser considerado una tendencia hacia el infinito (∞); aunque una ***N*** mucho más grande tal y como ***N=1,000,000*** o más podría ser elegida). Por tanto, nos posicionamos en la **celda A21** y escribimos lo siguiente:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21				
22				
23				

=SEQUENCE(100000,1,1,1)

Esto significa que habrá **100,000 Filas**
En **1 Columna**
Comenzado **desde 1**
Incrementándose en 1 (desde 1 hasta llegar a N=100,000)

Por tanto, se verá como sigue:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
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5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			
100018	99,998			
100019	99,999			
100020	100,000			

5) Ahora, en la **celda B21**, escribimos lo siguiente:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			
100018	99,998			
100019	99,999			
100020	100,000			

=(B10)-((B12)*(A21))

Esta parte corresponde a la ecuación (M1) en rojo:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \text{……}(M1)$$

6) Se verá como sigue (luego de autoellenar hasta ***k=100,000***):

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999		
22	2	0.99998		
23	3	0.99997		
24	4	0.99996		
25	5	0.99995		
100016	99,996	0.00004		
100017	99,997	0.00003		
100018	99,998	0.00002		
100019	99,999	0.00001		
100020	100,000	0.00000		

7) En la **celda C21**, escribimos lo siguiente:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999		
22	2	0.99998		
23	3	0.99997		
24	4	0.99996		
25	5	0.99995		
100016	99,996	0.00004		
100017	99,997	0.00003		
100018	99,998	0.00002		
100019	99,999	0.00001		
100020	100,000	0.00000		

=720*B21

Esta parte corresponde a la ecuación (M1) en rojo:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \text{……}(M1)$$

8) Se verá como sigue (luego de autorellenar hasta ***k=100,000***):

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
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7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	
22	2	0.99998	719.98560	
23	3	0.99997	719.97840	
24	4	0.99996	719.97120	
25	5	0.99995	719.96400	
100016	99,996	0.00004	0.02880	
100017	99,997	0.00003	0.02160	
100018	99,998	0.00002	0.01440	
100019	99,999	0.00001	0.00720	
100020	100,000	0.00000	0.00000	

9) En la **celda D21**, escribimos lo siguiente:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
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7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	
22	2	0.99998	719.98560	
23	3	0.99997	719.97840	
24	4	0.99996	719.97120	
25	5	0.99995	719.96400	
100016	99,996	0.00004	0.02880	
100017	99,997	0.00003	0.02160	
100018	99,998	0.00002	0.01440	
100019	99,999	0.00001	0.00720	
100020	100,000	0.00000	0.00000	

=C21*(B12)

Esta parte corresponde a la ecuación (M1) en rojo:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \text{……}(M1)$$

10) Se verá como sigue (luego de autorellenar hasta ***k=100,000***):

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	0.0071999
22	2	0.99998	719.98560	0.0071999
23	3	0.99997	719.97840	0.0071998
24	4	0.99996	719.97120	0.0071997
25	5	0.99995	719.96400	0.0071996
100016	99,996	0.00004	0.02880	0.0000003
100017	99,997	0.00003	0.02160	0.0000002
100018	99,998	0.00002	0.01440	0.0000001
100019	99,999	0.00001	0.00720	0.0000001
100020	100,000	0.00000	0.00000	0.0000000
100021				
100022				
100023				
100024				

11) En la **celda C100021**, escribimos **"SUMA"**:

12) En la **celda D100021**, escribimos:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
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17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	0.0071999
22	2	0.99998	719.98560	0.0071999
23	3	0.99997	719.97840	0.0071998
24	4	0.99996	719.97120	0.0071997
25	5	0.99995	719.96400	0.0071996
100016	99,996	0.00004	0.02880	0.0000003
100017	99,997	0.00003	0.02160	0.0000002
100018	99,998	0.00002	0.01440	0.0000001
100019	99,999	0.00001	0.00720	0.0000001
100020	100,000	0.00000	0.00000	0.0000000
100021			SUMA	
100022				
100023				
100024				

=SUM(D21:D100020)

Esta parte corresponde a la ecuación (M1) en rojo:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \text{ ……} (M1)$$

13) Se verá como sigue:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	0.0071999
22	2	0.99998	719.98560	0.0071999
23	3	0.99997	719.97840	0.0071998
24	4	0.99996	719.97120	0.0071997
25	5	0.99995	719.96400	0.0071996
100016	99,996	0.00004	0.02880	0.0000003
100017	99,997	0.00003	0.02160	0.0000002
100018	99,998	0.00002	0.01440	0.0000001
100019	99,999	0.00001	0.00720	0.0000001
100020	100,000	0.00000	0.00000	0.0000000
100021			SUMA	360
100022				
100023				
100024				
100025				

14) En la **celda C100023**, escribimos:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \cdots\cdots (M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	0.0071999
22	2	0.99998	719.98560	0.0071999
23	3	0.99997	719.97840	0.0071998
24	4	0.99996	719.97120	0.0071997
25	5	0.99995	719.96400	0.0071996
100016	99,996	0.00004	0.02880	0.0000003
100017	99,997	0.00003	0.02160	0.0000002
100018	99,998	0.00002	0.01440	0.0000001
100019	99,999	0.00001	0.00720	0.0000001
100020	100,000	0.00000	0.00000	0.0000000
100021			SUMA	360
100022				
100023			f'''(x_0)	=B13
100024		(-)	SUMATORIA	=D100021
100025		(=)	C1	=B13-D100021

De acuerdo a:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \cdots\cdots (M1)$$

Finalmente:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	0.0071999
22	2	0.99998	719.98560	0.0071999
23	3	0.99997	719.97840	0.0071998
24	4	0.99996	719.97120	0.0071997
25	5	0.99995	719.96400	0.0071996
100016	99,996	0.00004	0.02880	0.0000003
100017	99,997	0.00003	0.02160	0.0000002
100018	99,998	0.00002	0.01440	0.0000001
100019	99,999	0.00001	0.00720	0.0000001
100020	100,000	0.00000	0.00000	0.0000000
100021			SUMA	360
100022				
100023			f'''(x_0)	360
100024		(-)	SUMATORIA	360
100025		(=)	C1	0

$\therefore C_1 = 0$

VERIFICANDO EL VALOR DE LA CONSTANTE:

$$\rightarrow \frac{d^3}{dx^3}y = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n} f\left[x-\left(\frac{x}{n}\right).(k)\right]\right\}.\left(\frac{x}{n}\right)\right\} + C_1 \quad \cdots\cdots(m1A)$$

Condición Inicial (x,y,y',y'',y''')=(1,14; 36,126,360)

Sol…

$$\rightarrow 360 = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n} 720\left[1-\left(\frac{1}{n}\right).(k)\right]\right\}.\left(\frac{1}{n}\right)\right\} + C_1$$

$$360 = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n}\left[720-720\left(\frac{1}{n}\right).(k)\right]\right\}.\left(\frac{1}{n}\right)\right\} + C_1$$

$$360 = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n}\left[\frac{720}{n}-720\left(\frac{1}{n^2}\right).(k)\right]\right\}\right\} + C_1$$

$$360 = \left\{\lim_{n\to\infty}\left[\frac{720}{n}\sum_{k=1}^{n}1-720\left(\frac{1}{n^2}\right)\sum_{k=1}^{n}k\right]\right\} + C_1$$

$$360 = \left\{\lim_{n\to\infty}\left[\frac{720}{n}(n)-720\left(\frac{1}{n^2}\right)\left(\frac{n^2+n}{2}\right)\right]\right\} + C_1$$

$$360 = \left\{\lim_{n\to\infty}\left[720-360\left(\frac{n^2+n}{n^2}\right)\right]\right\} + C_1$$

$$360 = \left\{\lim_{n\to\infty}\left[720-360\left(1+\frac{1}{n}\right)\right]\right\} + C_1$$

$$360 = \left\{\lim_{n\to\infty}\left(720-360-360\frac{1}{n}\right)\right\} + C_1$$

$$360 = \left(\lim_{n\to\infty}360-360\lim_{n\to\infty}\frac{1}{n}\right) + C_1$$

$$360 = (360-0) + C_1$$

$$360 = 360 + C_1$$

$$\therefore C_1 = 0 \ \blacksquare$$

II. SEGUNDO PASO A LA IZQUIERDA

Vamos a dar el ***Segundo paso a la izquierda***, por tanto ***m=2.***

	$\lim_{n\to\infty}\left\{\frac{f\left[x-(2)\left(\frac{x}{n}\right)\right]}{0!}+2\frac{f\left[x-(2+1)\left(\frac{x}{n}\right)\right]}{1!}+\frac{(2)(2+1)}{2!}.f\left[x-(2+2)\left(\frac{x}{n}\right)\right]\right.$ $+\frac{(2)(2+1)(2+2)}{3!}.f\left[x-(2+3)\left(\frac{x}{n}\right)\right]$ $+\frac{(2)(2+1)(2+2)(2+3)}{4!}.f\left[x-(2+4)\left(\frac{x}{n}\right)\right]$ $+\frac{(2)(2+1)(2+2)(2+3)(2+4)}{5!}.f\left[x-(2+5)\left(\frac{x}{n}\right)\right]$ $\left.+\cdots\right\}.\left(\frac{x}{n}\right)^2$	$\cdots(\boldsymbol{\Omega 1})$
(+)	$\sum_{k=1}^{2}\left[C_k.\frac{x^{(2-k)}}{(2-k)!}\right]$	
(=)	$\boldsymbol{\frac{d^{(4-2)}}{dx^{(4-2)}}y}$	

	$\lim_{n\to\infty}\left\{f\left[x-2\left(\frac{x}{n}\right)\right]+2f\left[x-3\left(\frac{x}{n}\right)\right]+3f\left[x-4\left(\frac{x}{n}\right)\right]+4f\left[x-5\left(\frac{x}{n}\right)\right]+5f\left[x-6\left(\frac{x}{n}\right)\right]\right.$ $\left.+6f\left[x-7\left(\frac{x}{n}\right)\right]+\cdots\right\}.\left(\frac{x}{n}\right)^2$
(+)	$\left[C_1.\frac{x^{(2-1)}}{(2-1)!}\right]+\left[C_2.\frac{x^{(2-2)}}{(2-2)!}\right]$
(=)	$\boldsymbol{\frac{d^2}{dx^2}y}$

	$\lim_{n\to\infty}\left\{f\left[x-2\left(\frac{x}{n}\right)\right]+2f\left[x-3\left(\frac{x}{n}\right)\right]+3f\left[x-4\left(\frac{x}{n}\right)\right]+4f\left[x-5\left(\frac{x}{n}\right)\right]+5f\left[x-6\left(\frac{x}{n}\right)\right]\right.$ $\left.+6f\left[x-7\left(\frac{x}{n}\right)\right]+\cdots\right\}.\left(\frac{x}{n}\right)^2$
(+)	C_1x+C_2
(=)	$\boldsymbol{\frac{d^2}{dx^2}y}$

Por tanto:

$$\boldsymbol{\frac{d^2}{dx^2}y=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-1}k.f\left[x-(k+1).\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)^2\right\}+C_1\,x+C_2}$$

$$\boldsymbol{\frac{d^2}{dx^2}y=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-1}k.f\left[x-(k+1).\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)^2\right\}+C_1\,x+C_2\cdots\cdots\cdots\cdots\cdots\cdots\cdots(m2B)}$$

La expresión en ROJO representa la *Segunda Integral Indefinida*

Sin embargo, dado que la Hoja de Cálculo será usada para encontrar numéricamente la ***Función Original***, la última expresión de arriba **[*m2B*]** se convierte en lo siguiente (al elegir un número muy grande ***N*** que tienda al infinito):

$$f''(x_0) \cong \left\{\left\{\sum_{k=1}^{N-1} k.f\left[x_0 - \left(\frac{x_0}{N}\right).(k+1)\right]\right\}.\left(\frac{x_0}{N}\right)^2\right\} + C_1\,(x_0) + C_2 \cdots (m2)$$

Resolviendo para C_2, tenemos:

$$\rightarrow C_2 \cong f''(x_0) - \left\{\left\{\sum_{k=1}^{N-1} k.f\left[x_0 - \left(\frac{x_0}{N}\right).(k+1)\right]\right\}.\left(\frac{x_0}{N}\right)^2\right\} - C_1\,(x_0) \cdots (m2)$$

La función derivada es:

$$\rightarrow \frac{d^4}{dx^4} = 720x$$

Condición Inicial (x,y,y',y'',y''')=(1,14; 36,126,360)

Infinito Elegido Numéricamente: N=100,000

Por tanto, ***la ecuación (m2)*** se convierte en:

$$\rightarrow C_2 \cong f''(x_0) - \left\{\left\{\sum_{k=1}^{N-1} k.f\left[x_0 - \left(\frac{x_0}{N}\right).(k+1)\right]\right\}.\left(\frac{x_0}{N}\right)^2\right\} - C_1\,(x_0) \cdots (m2)$$

$$C_2 \cong 126 - \left\{\left\{\sum_{k=1}^{100{,}000-1} 720k\left[1 - \left(\frac{1}{100{,}000}\right).(k+1)\right]\right\}.\left(\frac{1}{100{,}000}\right)^2\right\} - C_1\,(1) \cdots (m2)$$

$$C_2 \cong 126 - \left\{\left\{\sum_{k=1}^{99{,}999} 720k\left[1 - \left(\frac{1}{100{,}000}\right).(k+1)\right]\right\}.\left(\frac{1}{100{,}000}\right)^2\right\} - C_1 \cdots (M2)$$

CALCULANDO M=2 EN HOJA DE CÁLCULO

Abrimos otra Hoja de Cálculo y la nombramos ***M=2***

La ecuación ***(M2)*** es:

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$$

1) Escribimos esta ecuación en la **celda A1**:

2) En la **celda A10**, comenzamos a escribir lo siguiente:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	=(B10/B11)		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20				
21				

3) En la **celda A20**, comenzamos a poner lo siguiente:

	A	B	C	D
1	$C_2 \cong 126 - \left\{\sum_{k=1}^{99,999}\left\{720k\left[1-\left(\frac{1}{100,000}\right).(k+1)\right].\left(\frac{1}{100,000}\right)^2\right\}\right\} - (C_1)$ ……$(M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0-\left(\frac{x_0}{N}\right)(k+1)\right]$	$720k\left[x_0-\left(\frac{x_0}{N}\right).(k+1)\right]$	$\left\{720k\left[x_0-\left(\frac{x_0}{N}\right).(k+1)\right].\left(\frac{x_0}{N}\right)^2\right\}$
21				
22				
23				

De acuerdo a:

$$C_2 \cong f''(x_0) - \left\{\sum_{k=1}^{N-1}\left\{k.f\left[x_0-\left(\frac{x_0}{N}\right).(k+1)\right].\left(\frac{x_0}{N}\right)^2\right\}\right\} - (C_1)(x_0) \cdots\cdots (m2)$$

$\left[x_0-\left(\frac{x_0}{N}\right).(k+1)\right]$ Representa la ***x modficada*** en **(M2)**.

$f\left[x_0-\left(\frac{x_0}{N}\right).(k+1)\right]$ Representa esa ***x modificada*** sustituida en la función ***f(x)***

4) De acuerdo a la ecuación **(M2)**, el rango de ***k*** va de ***k=1*** a ***k= 99,999*** ya que la ***N*** escogida para este caso es igual a ***N=100,000*** (un número muy grande para ser considerado una tendencia hacia el infinito (∞); aunque una ***N*** mucho más grande tal y como ***N=1,000,000*** o más podría ser elegida). Por tanto, nos posicionamos en la **celda A21** y escribimos lo siguiente:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21				
22				
23				

=SEQUENCE(100000-1,1,1,1)

Esto significa que habrá **99,999 Filas**
En **1 Columna**
Comenzando **desde 1**
Incrementándose en 1 (desde 1 hasta llegar a N-1=99,999)

Por tanto, se verá como sigue:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			
100018	99,998			
100019	99,999			

5) Ahora, en la **celda B21**, escribimos lo siguiente:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			
100018	99,998			
100019	99,999			

=(B10)-((B12)*(A21+1))

Esta parte corresponde a la ecuación (M2) en rojo:

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$$

6) Se verá como sigue (luego de autoellenar hasta ***k=99,999***):

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998		
22	2	0.99997		
23	3	0.99996		
24	4	0.99995		
25	5	0.99994		
100016	99,996	0.00003		
100017	99,997	0.00002		
100018	99,998	0.00001		
100019	99,999	0.00000		

7) En la **celda C21**, escribimos lo siguiente:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998		
22	2	0.99997		
23	3	0.99996		
24	4	0.99995		
25	5	0.99994		
100016	99,996	0.00003		
100017	99,997	0.00002		
100018	99,998	0.00001		
100019	99,999	0.00000		

=720*A21*B21

Esta parte corresponde a la ecuación (M2) en rojo:

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$$

8) Se verá como sigue (luego de autorellenar hasta ***k=99,999***):

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	
22	2	0.99997	1,439.95680	
23	3	0.99996	2,159.91360	
24	4	0.99995	2,879.85600	
25	5	0.99994	3,599.78400	
100016	99,996	0.00003	2,159.91360	
100017	99,997	0.00002	1,439.95680	
100018	99,998	0.00001	719.98560	
100019	99,999	0.00000	0.00000	

9) En la **celda D21**, escribimos lo siguiente:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	
22	2	0.99997	1,439.95680	
23	3	0.99996	2,159.91360	
24	4	0.99995	2,879.85600	
25	5	0.99994	3,599.78400	
100016	99,996	0.00003	2,159.91360	
100017	99,997	0.00002	1,439.95680	
100018	99,998	0.00001	719.98560	
100019	99,999	0.00000	0.00000	

=C21*(B12)^2

Esta parte corresponde a la ecuación (M2) en rojo:

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$$

10) Se verá como sigue (luego de autorellenar hasta ***k=99,999***):

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	0.00000007
22	2	0.99997	1,439.95680	0.00000014
23	3	0.99996	2,159.91360	0.00000022
24	4	0.99995	2,879.85600	0.00000029
25	5	0.99994	3,599.78400	0.00000036
100016	99,996	0.00003	2,159.91360	0.00000022
100017	99,997	0.00002	1,439.95680	0.00000014
100018	99,998	0.00001	719.98560	0.00000007
100019	99,999	0.00000	0.00000	0.00000000

11) En la **celda C100021**, escribimos **"SUMA"**:

12) En la **celda D100021**, escribimos:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	0.00000007
22	2	0.99997	1,439.95680	0.00000014
23	3	0.99996	2,159.91360	0.00000022
24	4	0.99995	2,879.85600	0.00000029
25	5	0.99994	3,599.78400	0.00000036
100016	99,996	0.00003	2,159.91360	0.00000022
100017	99,997	0.00002	1,439.95680	0.00000014
100018	99,998	0.00001	719.98560	0.00000007
100019	99,999	0.00000	0.00000	0.00000000
100020				
100021			SUMA	
100022				
100023				
100024				

=SUM(D21:D100019)

Esta parte corresponde a la ecuación (M2) en rojo:

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$$

13) Se verá como sigue:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	0.00000007
22	2	0.99997	1,439.95680	0.00000014
23	3	0.99996	2,159.91360	0.00000022
24	4	0.99995	2,879.85600	0.00000029
25	5	0.99994	3,599.78400	0.00000036
100016	99,996	0.00003	2,159.91360	0.00000022
100017	99,997	0.00002	1,439.95680	0.00000014
100018	99,998	0.00001	719.98560	0.00000007
100019	99,999	0.00000	0.00000	0.00000000
100021			SUMA	119.9964

14) En la **celda C100023**, escribimos:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	0.00000007
22	2	0.99997	1,439.95680	0.00000014
23	3	0.99996	2,159.91360	0.00000022
24	4	0.99995	2,879.85600	0.00000029
25	5	0.99994	3,599.78400	0.00000036
100016	99,996	0.00003	2,159.91360	0.00000022
100017	99,997	0.00002	1,439.95680	0.00000014
100018	99,998	0.00001	719.98560	0.00000007
100019	99,999	0.00000	0.00000	0.00000000
100020				
100021			SUMA	119.9964
100022				
100023			f''(x_0)	=B13
100024			(-) SUMATORIA	=D100021
100025			(-) C1(calculada previamente)	0
100026			(=) C2	=B13-D100021-D100025

De acuerdo a:

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$$

Finalmente:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	0.00000007
22	2	0.99997	1,439.95680	0.00000014
23	3	0.99996	2,159.91360	0.00000022
24	4	0.99995	2,879.85600	0.00000029
25	5	0.99994	3,599.78400	0.00000036
100016	99,996	0.00003	2,159.91360	0.00000022
100017	99,997	0.00002	1,439.95680	0.00000014
100018	99,998	0.00001	719.98560	0.00000007
100019	99,999	0.00000	0.00000	0.00000000
100020				
100021			SUMA	119.9964
100022				
100023			f''(x_0)	126
100024		(-)	SUMATORIA	119.9964
100025		(-)	C1(calculada previamente)	0
100026		(=)	C2	6.0036

$\therefore C_2 = 6.0036$

VERIFICANDO EL VALOR DE LA CONSTANTE:

$$\rightarrow \frac{d^2}{dx^2}y = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-1} k.f\left[x-(k+1).\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)^2\right\} + C_1\,x + C_2 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (m2B)$$

Condición Inicial (x,y,y',y'',y''')=(1,14; 36,126,360)

La verificación previa mostró que C_1=0:

Sol…

$$126 = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-1} 720k\left[1-(k+1).\left(\frac{1}{n}\right)\right]\right\}.\left(\frac{1}{n}\right)^2\right\} + (0)(1) + C_2$$

$$126 = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-1} 720k\left[1-(k+1).\left(\frac{1}{n}\right)\right]\right\}.\left(\frac{1}{n}\right)^2\right\} + C_2$$

$$126 = \lim_{n\to\infty}\left\{\sum_{k=1}^{n-1} 720k\left[\left(\frac{1}{n}\right)^2-(k+1).\left(\frac{1}{n}\right).\left(\frac{1}{n}\right)^2\right]\right\} + C_2$$

$$126 = \lim_{n\to\infty}\left\{\sum_{k=1}^{n-1} 720k\left[\left(\frac{1}{n}\right)^2-(k+1).\left(\frac{1}{n}\right)^3\right]\right\} + C_2$$

$$126 = \lim_{n\to\infty}\left\{\sum_{k=1}^{n-1} 720k\left[\left(\frac{1}{n}\right)^2-(k+1).\left(\frac{1}{n}\right)^3\right]\right\} + C_2$$

$$126 = \lim_{n\to\infty}\left\{\sum_{k=1}^{n-1} 720k\left\{\left(\frac{1}{n}\right)^2-\left[k\left(\frac{1}{n}\right)^3+\left(\frac{1}{n}\right)^3\right]\right\}\right\} + C_2$$

$$126 = \lim_{n\to\infty}\left\{\sum_{k=1}^{n-1}\left[\left(\frac{720}{n^2}\right)k-720\left(\frac{k^2}{n^3}\right)-\left(\frac{720}{n^3}\right)k\right]\right\} + C_2$$

$$126 = \lim_{n\to\infty}\left[\left(\frac{720}{n^2}\right)\sum_{k=1}^{n-1}k-\frac{720}{n^3}\sum_{k=1}^{n-1}k^2-\left(\frac{720}{n^3}\right)\sum_{k=1}^{n-1}k\right] + C_2$$

$$126 = \lim_{n\to\infty}\left[\left(\frac{720}{n^2}\right)\left(\frac{n^2-n}{2}\right)-\left(\frac{720}{n^3}\right)\left(\frac{2n^3-3n^2+n}{6}\right)-\left(\frac{720}{n^3}\right)\left(\frac{n^2-n}{2}\right)\right] + C_2$$

$$126 = \lim_{n\to\infty}\left[360\left(\frac{n^2-n}{n^2}\right)-120\left(\frac{2n^3-3n^2+n}{n^3}\right)-360\left(\frac{n^2-n}{n^3}\right)\right] + C_2$$

$$126 = \lim_{n\to\infty}\left[360\left(1-\frac{1}{n}\right)-120\left(2-\frac{3}{n}+\frac{1}{n^2}\right)-360\left(\frac{1}{n}-\frac{1}{n^2}\right)\right] + C_2$$

$$126 = \left[360\lim_{n\to\infty}\left(1-\frac{1}{n}\right)-120\lim_{n\to\infty}\left(2-\frac{3}{n}+\frac{1}{n^2}\right)-360\lim_{n\to\infty}\left(\frac{1}{n}-\frac{1}{n^2}\right)\right] + C_2$$

$$126 = \left[360\left(\lim_{n\to\infty} 1 - \lim_{n\to\infty}\frac{1}{n}\right) - 120\left(\lim_{n\to\infty} 2 - \lim_{n\to\infty}\frac{3}{n} + \lim_{n\to\infty}\frac{1}{n^2}\right) - 360\left(\lim_{n\to\infty}\frac{1}{n} - \lim_{n\to\infty}\frac{1}{n^2}\right)\right] + C_2$$

$$126 = [360(1-0) - 120(2-0-0) - 360(0-0)] + C_2$$

$$126 = [360(1) - 120(2) - 360(0)] + C_2$$

$$126 = 360 - 240 - 0 + C_2$$

$$126 = 120 + C_2$$

$$\therefore C_2 = 6 \blacksquare$$

III. TERCER PASO A LA IZQUIERDA

Vamos a dar el ***Tercer paso a la izquierda***, por tanto ***m=3.***

	$\lim_{n\to\infty}\left\{\frac{f\left[x-(3)\left(\frac{x}{n}\right)\right]}{0!}+(3).\frac{f\left[x-(3+1)\left(\frac{x}{n}\right)\right]}{1!}+\frac{(3)(3+1)}{2!}.f\left[x-(3+2)\left(\frac{x}{n}\right)\right]\right.$ $\left.+\frac{(3)(3+1)(3+2)}{3!}.f\left[x-(3+3)\left(\frac{x}{n}\right)\right]+\frac{(3)(3+1)(3+2)(3+3)}{4!}.f\left[x-(3+4)\left(\frac{x}{n}\right)\right]+\frac{(3)(3+1)(3+2)(3+3)(3+4)}{5!}.f\left[x-(3+5)\left(\frac{x}{n}\right)\right]+\cdots\right\}.\left(\frac{x}{n}\right)^3$	$\cdots(\boldsymbol{\Omega 1})$
(+)	$\sum_{k=1}^{3}\left[C_k.\frac{x^{(3-k)}}{(3-k)!}\right]$	
(=)	$\frac{\boldsymbol{d^{(4-3)}}}{\boldsymbol{dx^{(4-3)}}}\boldsymbol{y}$	

	$\lim_{n\to\infty}\left\{f\left[x-3\left(\frac{x}{n}\right)\right]+3\frac{f\left[x-4\left(\frac{x}{n}\right)\right]}{1!}+6f\left[x-5\left(\frac{x}{n}\right)\right]+10f\left[x-6\left(\frac{x}{n}\right)\right]+15f\left[x-7\left(\frac{x}{n}\right)\right]\right.$ $\left.+21f\left[x-8\left(\frac{x}{n}\right)\right]+\cdots\right\}.\left(\frac{x}{n}\right)^3$
(+)	$\left[C_1.\frac{x^{(3-1)}}{(3-1)!}\right]+\left[C_2.\frac{x^{(\mathbf{3}-2)}}{(3-2)!}\right]+\left[C_3.\frac{x^{(\mathbf{3}-3)}}{(3-3)!}\right]$
(=)	$\frac{\boldsymbol{d^1}}{\boldsymbol{dx^1}}\boldsymbol{y}$

	$\lim_{n\to\infty}\left\{f\left[x-3\left(\frac{x}{n}\right)\right]+3\frac{f\left[x-4\left(\frac{x}{n}\right)\right]}{1!}+6f\left[x-5\left(\frac{x}{n}\right)\right]+10f\left[x-6\left(\frac{x}{n}\right)\right]+15f\left[x-7\left(\frac{x}{n}\right)\right]\right.$ $\left.+21f\left[x-8\left(\frac{x}{n}\right)\right]+\cdots\right\}.\left(\frac{x}{n}\right)^3$
(+)	$\frac{1}{2}C_1x^2+C_2x+C_3$
(=)	$\frac{\boldsymbol{d^1}}{\boldsymbol{dx^1}}\boldsymbol{y}$

Por tanto:

$$\frac{d^1}{dx^1}y=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-2}\left(\frac{k^2+k}{2}\right).f\left[x-(k+2).\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)^3\right\}+\frac{1}{2}C_1\,x^2+C_2x+C_3$$

$$\frac{d^1}{dx^1}y=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-2}\left(\frac{k^2+k}{2}\right).f\left[x-(k+2).\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)^3\right\}+\frac{1}{2}C_1\,x^2+C_2x+C_3\cdots\cdots(m3C)$$

La expresión en ROJO representa la *Tercera Integral Indefinida*

Sin embargo, dado que la Hoja de Cálculo será usada para encontrar numéricamente la ***Función Original***, la última expresión de arriba **[*m3C*]** se convierte en lo siguiente (al elegir un número muy grande ***N*** que tienda al infinito):

$$f'(x_0)\cong\left\{\left\{\sum_{k=1}^{N-2}\left(\frac{k^2+k}{2}\right).f\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]\right\}.\left(\frac{x_0}{N}\right)^3\right\}+\frac{1}{2}C_1\,(x_0)^2+C_2(x_0)+C_3\cdots\cdots(m3)$$

Resolviendo para C_3, tenemos:

$$\rightarrow C_3\cong f'(x_0)-\left\{\left\{\sum_{k=1}^{N-2}\left(\frac{k^2+k}{2}\right).f\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]\right\}.\left(\frac{x_0}{N}\right)^3\right\}-\frac{1}{2}C_1\,(x_0)^2-C_2(x_0)\cdots\cdots(m3)$$

La función derivada es:

$$\rightarrow\frac{d^4}{dx^4}=720x$$

Condición Inicial (x,y,y',y'',y''')=(1,14; 36,126,360)

Infinito Elegido Numéricamente: N=100,000

Por tanto, ***la ecuación (m3)*** se convierte en:

$$\rightarrow C_3\cong f'(x_0)-\left\{\left\{\sum_{k=1}^{N-2}\left(\frac{k^2+k}{2}\right).f\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]\right\}.\left(\frac{x_0}{N}\right)^3\right\}-\frac{1}{2}C_1\,(x_0)^2-C_2(x_0)\cdots\cdots(m3)$$

$$C_3\cong 36-\left\{\left\{\sum_{k=1}^{100000-2}720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right]\right\}.\left(\frac{1}{100,000}\right)^3\right\}-\frac{1}{2}C_1\,(1)^2-C_2(1)\cdots\cdots(m3)$$

$$C_3\cong 36-\left\{\left\{\sum_{k=1}^{99,998}720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right]\right\}.\left(\frac{1}{100,000}\right)^3\right\}-\frac{1}{2}C_1-C_2\cdots\cdots(M3)$$

CALCULANDO M=3 EN HOJA DE CÁLCULO

Abrimos otra Hoja de Cálculo y la nombramos ***M=3***

La ecuación ***(M3)*** es:

$$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720 \left(\frac{k^2+k}{2} \right) \left[1 - \left(\frac{1}{100,000} \right) . (k+2) \right] . \left(\frac{1}{100,000} \right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$$

1) Escribimos esta ecuación en la **celda A1**:

2) En la **celda A10**, comenzamos a escribir lo siguiente:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720 \left(\frac{k^2+k}{2} \right) \left[1 - \left(\frac{1}{100,000} \right) . (k+2) \right] . \left(\frac{1}{100,000} \right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	=(B10/B11)		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20				
21				

3) En la **celda A20**, comenzamos a poner lo siguiente:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2)$ ……$(M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21				
22				
23				

De acuerdo a:

$$C_3 \cong f'(x_0) - \left\{ \sum_{k=1}^{N-2} \left\{ \left(\frac{k^2+k}{2}\right).f\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3 \right\} \right\} - \frac{1}{2}(C_1)(x_0)^2 - (C_2)(x_0)) \text{ ……} (m3)$$

$\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right]$ Representa la ***x modificada*** en **(M3)**.

$f\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right]$ Representa esa ***x modificada*** sustituida en la función ***f(x)***

4) De acuerdo a la ecuación **(M3)**, el rango de ***k*** va de ***k=1*** a ***k= 99,998*** ya que la ***N*** escogida para este caso es igual a ***N=100,000*** (un número muy grande para ser considerado una tendencia hacia el infinito (∞); aunque una ***N*** mucho más grande tal y como ***N=1,000,000*** o más podría ser elegida). Por tanto, nos posicionamos en la **celda A21** y escribimos lo siguiente:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21				
22				
23				

=SEQUENCE(100000-2,1,1,1)

Esto significa que habrá **99,998 Filas**
En **1 Columna**
Comenzando **desde 1**
Incrementándose en 1 (desde 1 hasta llegar a N-2=99,998)

Por tanto, se verá como sigue:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2)$ ……$(M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			
100018	99,998			

5) Ahora, en la **celda B21**, escribimos lo siguiente:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720 \left(\frac{k^2+k}{2} \right) \left[1 - \left(\frac{1}{100,000} \right) \cdot (k+2) \right] \cdot \left(\frac{1}{100,000} \right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+2) \right]$	$720 \left(\frac{k^2+k}{2} \right) \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+2) \right]$	$\left\{ 720 \left(\frac{k^2+k}{2} \right) \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+2) \right] \cdot \left(\frac{x_0}{N} \right)^3 \right\}$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			
100018	99,998			

=(B10)-((B12)*(A21+2))

Esta parte corresponde a la ecuación (M3) en rojo:

$$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720 \left(\frac{k^2+k}{2} \right) \left[1 - \left(\frac{1}{100,000} \right) \cdot (k+2) \right] \cdot \left(\frac{1}{100,000} \right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$$

6) Se verá como sigue (luego de autoellenar hasta ***k=99,998***):

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2)$ ……$(M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997		
22	2	0.99996		
23	3	0.99995		
24	4	0.99994		
25	5	0.99993		
100016	99,996	0.00002		
100017	99,997	0.00001		
100018	99,998	0.00000		

7) En la **celda C21**, escribimos lo siguiente:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997		
22	2	0.99996		
23	3	0.99995		
24	4	0.99994		
25	5	0.99993		
100016	99,996	0.00002		
100017	99,997	0.00001		
100018	99,998	0.00000		

=720*(((A21^2+A21)/2))*B21

Esta parte corresponde a la ecuación (M3) en rojo:

$$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$$

8) Se verá como sigue (luego de autorellenar hasta ***k=99,998***):

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2)$ ……$(M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	
22	2	0.99996	2,159.91360	
23	3	0.99995	4,319.78400	
24	4	0.99994	7,199.56800	
25	5	0.99993	10,799.24400	
100016	99,996	0.00002	71,994,960.08607	
100017	99,997	0.00001	35,998,200.02144	
100018	99,998	0.00000	0.00000	

9) En la **celda D21**, escribimos lo siguiente:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	
22	2	0.99996	2,159.91360	
23	3	0.99995	4,319.78400	
24	4	0.99994	7,199.56800	
25	5	0.99993	10,799.24400	
100016	99,996	0.00002	71,994,960.08607	
100017	99,997	0.00001	35,998,200.02144	
100018	99,998	0.00000	0.00000	

=C21*(B12)^3

Esta parte corresponde a la ecuacioón (M3) en rojo:

$$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$$

10) Se verá como sigue (luego de autorellenar hasta ***k=99,998***):

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2)$ ·····$(M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0-\left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	0.00000
22	2	0.99996	2,159.91360	0.00000
23	3	0.99995	4,319.78400	0.00000
24	4	0.99994	7,199.56800	0.00000
25	5	0.99993	10,799.24400	0.00000
100016	99,996	0.00002	71,994,960.08607	0.00000
100017	99,997	0.00001	35,998,200.02144	0.00000
100018	99,998	0.00000	0.00000	0.00000

11) En la **celda C100021**, escribimos **"SUMA"**:

12) En la **celda D100021**, escribimos:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	0.00000
22	2	0.99996	2,159.91360	0.00000
23	3	0.99995	4,319.78400	0.00000
24	4	0.99994	7,199.56800	0.00000
25	5	0.99993	10,799.24400	0.00000
100016	99,996	0.00002	71,994,960.08607	0.00000
100017	99,997	0.00001	35,998,200.02144	0.00000
100018	99,998	0.00000	0.00000	0.00000
100019				
100020				
100021			SUMA	
100022				
100023				
100024				

=SUM(D21:D100018)

Esta parte corresponde a la ecuación (M3) en rojo:

$$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$$

13) Se verá como sigue:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2)$ ……$(M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0-\left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	0.00000
22	2	0.99996	2,159.91360	0.00000
23	3	0.99995	4,319.78400	0.00000
24	4	0.99994	7,199.56800	0.00000
25	5	0.99993	10,799.24400	0.00000
100016	99,996	0.00002	71,994,960.08607	0.00000
100017	99,997	0.00001	35,998,200.02144	0.00000
100018	99,998	0.00000	0.00000	0.00000
100021			SUMA	29.9982

14) En la **celda C100023**, escribimos:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	0.00000
22	2	0.99996	2,159.91360	0.00000
23	3	0.99995	4,319.78400	0.00000
24	4	0.99994	7,199.56800	0.00000
25	5	0.99993	10,799.24400	0.00000
100016	99,996	0.00002	71,994,960.08607	0.00000
100017	99,997	0.00001	35,998,200.02144	0.00000
100018	99,998	0.00000	0.00000	0.00000
100021			**SUMA**	**29.9982**
100022				
100023			**f'(x_0)**	=B13
100024			**(-) SUMATORIA**	=D100021
100025			**(-) (1/2)C1**(calculada previamente)	**=(1/2)*0**
100026			**(-) C2**(calculada previamente)	**6.0036**
100027			**(=) C3**	=B13-D100021-D100025-D100026

De acuerdo a:

$$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$$

Finalmente:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2)$ ……$(M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	0.00000
22	2	0.99996	2,159.91360	0.00000
23	3	0.99995	4,319.78400	0.00000
24	4	0.99994	7,199.56800	0.00000
25	5	0.99993	10,799.24400	0.00000
100016	99,996	0.00002	71,994,960.08607	0.00000
100017	99,997	0.00001	35,998,200.02144	0.00000
100018	99,998	0.00000	0.00000	0.00000
100021			SUMA	29.9982
100022				
100023			f'(x_0)	36
100024		(-)	SUMATORIA	29.9982
100025		(-)	(1/2)C1(calculada previamente)	0
100026		(-)	C2(calculada previamente)	6.0036
100027		(=)	C3	-0.0018

$$\therefore C_3 = -0.0018$$

VERIFICANDO EL VALOR DE LA CONSTANTE:

$$\rightarrow \frac{d^1}{dx^1}y = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-2}\left(\frac{k^2+k}{2}\right).f\left[x-(k+2).\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)^3\right\}+\frac{1}{2}C_1\,x^2+C_2x+C_3\cdots\cdots(m3C)$$

Condición Inicial (x,y,y',y'',y''')=(1,14; 36,126,360)

La verificación previa mostró que $C_1=0$ & $C_2=6$:

Sol…

$$36=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-2}720\left(\frac{k^2+k}{2}\right)\left[1-(k+2).\left(\frac{1}{n}\right)\right]\right\}.\left(\frac{1}{n}\right)^3\right\}+\frac{1}{2}(0)\,(1)^2+(6)(1)+C_3$$

$$36=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-2}720\left(\frac{k^2+k}{2}\right)\left[1-(k+2).\left(\frac{1}{n}\right)\right]\right\}.\left(\frac{1}{n}\right)^3\right\}+6+C_3$$

$$30=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-2}720\left(\frac{k^2+k}{2}\right)\left[1-(k+2).\left(\frac{1}{n}\right)\right]\right\}.\left(\frac{1}{n}\right)^3\right\}+C_3$$

$$30=\lim_{n\to\infty}\left\{\sum_{k=1}^{n-2}720\left(\frac{k^2+k}{2}\right)\left[\left(\frac{1}{n}\right)^3-(k+2).\left(\frac{1}{n}\right).\left(\frac{1}{n}\right)^3\right]\right\}+C_3$$

$$30=\lim_{n\to\infty}\left\{\sum_{k=1}^{n-2}360(k^2+k)\left[\frac{1}{n^3}-\frac{(k+2)}{n^4}\right]\right\}+C_3$$

$$30=\lim_{n\to\infty}\left\{\sum_{k=1}^{n-2}\left[\left(\frac{360k^2+360k}{n^3}\right)-\left(\frac{360k^3+1{,}080k^2+720k}{n^4}\right)\right]\right\}+C_3$$

$$30=\lim_{n\to\infty}\left[\frac{360}{n^3}\sum_{k=1}^{n-2}k^2+\frac{360}{n^3}\sum_{k=1}^{n-2}k-\frac{360}{n^4}\sum_{k=1}^{n-2}k^3-\frac{1{,}080}{n^4}\sum_{k=1}^{n-2}k^2-\frac{720}{n^4}\sum_{k=1}^{n-2}k\right]+C_3$$

$$\begin{aligned}30=\lim_{n\to\infty}\Bigg[&\left(\frac{360}{n^3}\right)\left(\frac{2n^3-3n^2+n}{6}\right)+\left(\frac{360}{n^3}\right)\left(\frac{n^2-n}{2}\right)\\&-\left(\frac{360}{n^4}\right)\left(\frac{n^4-6n^3+13n^2-12n+4}{4}\right)-\left(\frac{1{,}080}{n^4}\right)\left(\frac{2n^3-3n^2+n}{6}\right)\\&-\left(\frac{720}{n^4}\right)\left(\frac{n^2-n}{2}\right)\Bigg]+C_3\end{aligned}$$

$$\begin{aligned}30=\lim_{n\to\infty}\Bigg[&60\left(\frac{2n^3-3n^2+n}{n^3}\right)+180\left(\frac{n^2-n}{n^3}\right)-90\left(\frac{n^4-6n^3+13n^2-12n+4}{n^4}\right)\\&-180\left(\frac{2n^3-3n^2+n}{n^4}\right)-360\left(\frac{n^2-n}{n^4}\right)\Bigg]+C_3\end{aligned}$$

Aplicando límites:

$$\begin{aligned}30&=[60(2-0+0)+180(0-0)-90(1-0+0-0+0)-180(0-0+0)\\&\quad-360(0-0)]+C_3\\30&=[60(2)+180(0)-90(1)-180(0)-360(0)]+C_3\\30&=120-90+C_3\\30&=30+C_3\end{aligned}$$

$$\therefore C_3=0\ \blacksquare$$

IV. CUARTO PASO A LA IZQUIERDA

Vamos a dar el ***Cuarto paso a la izquierda***, por tanto ***m=4.***

	$\lim_{n\to\infty}\left\{\frac{f\left[x-(4)\left(\frac{x}{n}\right)\right]}{0!}+(4).\frac{f\left[x-(4+1)\left(\frac{x}{n}\right)\right]}{1!}+\frac{(4)(4+1)}{2!}.f\left[x-(4+2)\left(\frac{x}{n}\right)\right]\right.$ $+\frac{(4)(4+1)(4+2)}{3!}.f\left[x-(4+3)\left(\frac{x}{n}\right)\right]$ $+\frac{(4)(4+1)(4+2)(4+3)}{4!}.f\left[x-(4+4)\left(\frac{x}{n}\right)\right]$ $+\frac{(4)(4+1)(4+2)(4+3)(4+4)}{5!}.f\left[x-(4+5)\left(\frac{x}{n}\right)\right]$ $\left.+\cdots\right\}.\left(\frac{x}{n}\right)^4$	$\cdots(\boldsymbol{\Omega 1})$
(+)	$\sum_{k=1}^{4}\left[C_k.\frac{x^{(4-k)}}{(4-k)!}\right]$	
(=)	$\frac{\boldsymbol{d^{(4-4)}}}{\boldsymbol{dx^{(4-4)}}}\boldsymbol{y}$	

	$\lim_{n\to\infty}\left\{f\left[x-4\left(\frac{x}{n}\right)\right]+4f\left[x-5\left(\frac{x}{n}\right)\right]+10f\left[x-6\left(\frac{x}{n}\right)\right]+20f\left[x-7\left(\frac{x}{n}\right)\right]+35f\left[x-8\left(\frac{x}{n}\right)\right]\right.$ $\left.+56f\left[x-9\left(\frac{x}{n}\right)\right]+\cdots\right\}.\left(\frac{x}{n}\right)^4$
(+)	$\left[C_1.\frac{x^{(4-1)}}{(4-1)!}\right]+\left[C_2.\frac{x^{(4-2)}}{(4-2)!}\right]+\left[C_3.\frac{x^{(4-3)}}{(4-3)!}\right]+\left[C_4.\frac{x^{(4-4)}}{(4-4)!}\right]$
(=)	$\frac{\boldsymbol{d^0}}{\boldsymbol{dx^0}}\boldsymbol{y}$

	$\lim_{n\to\infty}\left\{f\left[x-4\left(\frac{x}{n}\right)\right]+4f\left[x-5\left(\frac{x}{n}\right)\right]+10f\left[x-6\left(\frac{x}{n}\right)\right]+20f\left[x-7\left(\frac{x}{n}\right)\right]+35f\left[x-8\left(\frac{x}{n}\right)\right]\right.$ $\left.+56f\left[x-9\left(\frac{x}{n}\right)\right]+\cdots\right\}.\left(\frac{x}{n}\right)^4$
(+)	$\frac{1}{6}C_1x^3+\frac{1}{2}C_2x^2+C_3x+C_4$
(=)	$\boldsymbol{y}$

Por tanto:

$$\boldsymbol{y=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-3}\left(\frac{k^3+3k^2+2k}{6}\right).f\left[x-(k+3).\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)^4\right\}+\frac{1}{6}C_1x^3+\frac{1}{2}C_2x^2+C_3x+C_4}$$

$$\boldsymbol{y=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-3}\left(\frac{k^3+3k^2+2k}{6}\right).f\left[x-(k+3).\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)^4\right\}+\frac{1}{6}C_1x^3+\frac{1}{2}C_2x^2+C_3x+C_4\cdots\cdots(m4D)}$$

La expresión en ROJO representa la *Cuarta Integral Indefinida*

Sin embargo, dado que la Hoja de Cálculo será usada para encontrar numéricamente la ***Función Original***, la última expresión de arriba **[*m4D*]** se convierte en lo siguiente (al elegir un número muy grande ***N*** que tienda al infinito):

$$f(x_0) \cong \left\{\left\{\sum_{k=1}^{N-3}\left(\frac{k^3+3k^2+2k}{6}\right).f\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\right\}+\frac{1}{6}C_1\,(x_0)^3+\frac{1}{2}C_2(x_0)^2+C_3(x_0)+C_4\cdots\cdots(m4)$$

Resolviendo para C_4, tenemos:

$$\rightarrow C_4 \cong f(x_0)-\left\{\left\{\sum_{k=1}^{N-3}\left(\frac{k^3+3k^2+2k}{6}\right).f\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\right\}-\frac{1}{6}C_1\,(x_0)^3-\frac{1}{2}C_2(x_0)^2-C_3(x_0)\cdots\cdots(m4)$$

La función derivada es:

$$\rightarrow \frac{d^4}{dx^4}=720x$$

Condición Inicial (x,y,y',y'',y''')=(1,14; 36,126,360)

Infinito Elegido Numéricamente: N=100,000

Por tanto, ***la ecuación (m4)*** se convierte en:

$$\rightarrow C_4 \cong f(x_0)-\left\{\left\{\sum_{k=1}^{N-3}\left(\frac{k^3+3k^2+2k}{6}\right).f\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\right\}-\frac{1}{6}C_1\,(x_0)^3-\frac{1}{2}C_2(x_0)^2-C_3(x_0)\cdots\cdots(m4)$$

$$C_4 \cong 14-\left\{\left\{\sum_{k=1}^{100{,}000-3}720\left(\frac{k^3+3k^2+2k}{6}\right)\left[1-\left(\frac{1}{100{,}000}\right).(k+3)\right]\right\}.\left(\frac{1}{100{,}000}\right)^4\right\}-\frac{1}{6}C_1\,(1)^3-\frac{1}{2}C_2(1)^2-C_3(1)\cdots\cdots(m4)$$

$$C_4 \cong 14-\left\{\left\{\sum_{k=1}^{99{,}997}720\left(\frac{k^3+3k^2+2k}{6}\right)\left[1-\left(\frac{1}{100{,}000}\right).(k+3)\right]\right\}.\left(\frac{1}{100{,}000}\right)^4\right\}-\frac{1}{6}C_1-\frac{1}{2}C_2-C_3\cdots\cdots(M4)$$

CALCULANDO M=4 EN HOJA DE CÁLCULO

Abrimos otra Hoja de Cálculo y la nombramos ***M=4***

La ecuación ***(M4)*** es:

$$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) . (k+3) \right] . \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$$

1) Escribimos esta ecuación en la **celda A1**:

2) En la **celda A10**, comenzamos a escribir lo siguiente:

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) . (k+3) \right] . \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	=(B10/B11)		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20				
21				

3) En la **celda A20**, comenzamos a poner lo siguiente:

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……(M4)			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right].\left(\frac{x_0}{N}\right)^4\right\}$
21				
22				
23				

De acuerdo a:

$$C_4 \cong f(x_0) - \left\{\sum_{k=1}^{N-3}\left\{\left[\frac{(k)(k+1)(k+2)}{6}\right].f\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right].\left(\frac{x_0}{N}\right)^4\right\}\right\} - \frac{1}{6}(C_1)(x_0)^3 - \frac{1}{2}(C_2)(x_0)^2 - (C_3)(x_0) \quad \ldots\ldots(m4)$$

$\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right]$ Representa la ***x modificada*** en **(M4)**.

$f\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right]$ Representa esa ***x modificada*** sustituida en la función ***f(x)***

4) De acuerdo a la ecuación **(M4)**, el rango de ***k*** va de ***k=1*** a ***k= 99,997*** ya que la ***N*** escogida para este caso es igual a ***N=100,000*** (un número muy grande para ser considerado una tendencia hacia el infinito (∞); aunque una ***N*** mucho más grande tal y como ***N=1,000,000*** o más podría ser elegida). Por tanto, nos posicionamos en la **celda A21** y escribimos lo siguiente:

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99{,}997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100{,}000} \right) \cdot (k+3) \right] \cdot \left(\frac{1}{100{,}000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+3) \right]$	$720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right]$	$\left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right] \cdot \left(\frac{x_0}{N} \right)^4 \right\}$
21				
22				
23				

=SEQUENCE(100000-3,1,1,1)

Esto significa que habrá **99,997 Filas**
En **1 Columna**
Comenzando **desde 1**
Incrementándose en 1 (desde 1 hasta llegar a N-3=99,997)

Por tanto, se verá como sigue:

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) \cdot (k+3) \right] \cdot \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+3) \right]$	$720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right]$	$\left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right] \cdot \left(\frac{x_0}{N} \right)^4 \right\}$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			

5) Ahora, en la **celda B21**, escribimos lo siguiente:

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……$(M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right].\left(\frac{x_0}{N}\right)^4\right\}$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			

=(B10)-((B12)*(A21+3))

Esta parte corresponde a la ecuación (M4) en rojo:

$$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$$

6) Se verá como sigue (luego de autoellenar hasta ***k=99,997***):

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) \cdot (k+3) \right] \cdot \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+3) \right]$	$720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right]$	$\left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right] \cdot \left(\frac{x_0}{N} \right)^4 \right\}$
21	1	0.99996		
22	2	0.99995		
23	3	0.99994		
24	4	0.99993		
25	5	0.99992		
100016	99,996	0.00001		
100017	99,997	0.00000		

7) En la **celda C21**, escribimos lo siguiente:

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……$(M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right].\left(\frac{x_0}{N}\right)^4\right\}$
21	1	0.99996		
22	2	0.99995		
23	3	0.99994		
24	4	0.99993		
25	5	0.99992		
100016	99,996	0.00001		
100017	99,997	0.00000		

=720*((((A21)*(A21+1)*(A21+2))/6))*B21

Esta parte corresponde a la ecuación (M4) en rojo:

$$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$$

8) Se verá como sigue (luego de autorellenar hasta ***k=99,997***):

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) \cdot (k+3) \right] \cdot \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+3) \right]$	$720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right]$	$\left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right] \cdot \left(\frac{x_0}{N} \right)^4 \right\}$
21	1	0.99996	719.97120	
22	2	0.99995	2,879.85600	
23	3	0.99994	7,199.56800	
24	4	0.99993	14,398.99200	
25	5	0.99992	25,197.98400	
100016	99,996	0.00001	1,199,892,003,114.51000	
100017	99,997	0.00000	0.00000	

9) En la **celda D21**, escribimos lo siguiente:

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) \cdot (k+3) \right] \cdot \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+3) \right]$	$720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right]$	$\left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right] \cdot \left(\frac{x_0}{N} \right)^4 \right\}$
21	1	0.99996	719.97120	
22	2	0.99995	2,879.85600	
23	3	0.99994	7,199.56800	
24	4	0.99993	14,398.99200	
25	5	0.99992	25,197.98400	
100016	99,996	0.00001	1,199,892,003,114.51000	
100017	99,997	0.00000	0.00000	

=C21*(B12)^4

Esta parte corresponde a la ecuación (M4) en rojo:

$$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) \cdot (k+3) \right] \cdot \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$$

10) Se verá como sigue (luego de autorellenar hasta ***k=99,997***):

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……$(M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000

11) En la **celda C100021**, escribimos **"SUMA"**:

12) En la **celda D100021**, escribimos:

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000
100018				
100019				
100020				
100021			SUMA	
100022				
100023				
100024				

=SUM(D21:D100017)

Esta parte corresponde a la ecuación (M4) en rojo:

$$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$$

13) Se verá como sigue:

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) \cdot (k+3) \right] \cdot \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+3) \right]$	$720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right]$	$\left\{ 720 \left(\frac{k^2+k}{2} \right) \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+2) \right] \cdot \left(\frac{x_0}{N} \right)^3 \right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000
100018				
100021			SUMA	5.9994

14) En la **celda C100023**, escribimos:

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……$(M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000
100021			SUMA	5.9994
100022				
100023			f(x_0)	=B13
100024			(-) SUMATORIA	=D100021
100025			(-) (1/6)C1(calculada previamente)	0
100026			(-) (1/2)C2(calculada previamente)	=(1/2)*(6.0036)
100027			(-) C3(calculada previamente)	-0.0018
100028			(=) C4	=B13-D100021-D100025-D100026-D100027

De acuerdo a:

$$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \text{ ……} (M4)$$

Finalmente:

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) \cdot (k+3) \right] \cdot \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+3) \right]$	$720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right]$	$\left\{ 720 \left(\frac{k^2+k}{2} \right) \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+2) \right] \cdot \left(\frac{x_0}{N} \right)^3 \right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000
100021			SUMA	5.9994
100022				
100023			f(x_0)	14
100024		(-)	SUMATORIA	5.9994
100025		(-)	(1/6)C1(calculada previamente)	0
100026		(-)	(1/2)C2(calculada previamente)	3.0018
100027		(-)	C3(calculada previamente)	-0.0018
100028		(=)	C4	5.0006

$$\therefore C_4 = 5.0006$$

VERIFICANDO EL VALOR DE LA CONSTANTE:

$$\rightarrow y=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-3}\left(\frac{k^3+3k^2+2k}{6}\right).f\left[x-(k+3).\left(\frac{x}{n}\right)\right]\right\}.\left(\frac{x}{n}\right)^4\right\}+\frac{1}{6}C_1\,x^3+\frac{1}{2}C_2x^2+C_3x+C_4\cdots\cdots(m4D)$$

Condición Inicial (x,y,y',y'',y''')=(1,14; 36,126,360)

La verificación previa mostró que $C_1=0$ & $C_2=6$ & $C_3=0$:

$$14=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-3}720\left(\frac{k^3+3k^2+2k}{6}\right)\left[1-(k+3).\left(\frac{1}{n}\right)\right]\right\}.\left(\frac{1}{n}\right)^4\right\}+\frac{1}{6}(0)(1)^3+\frac{1}{2}(6)(1)^2+(0)(1)+C_4$$

$$14=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-3}720\left(\frac{k^3+3k^2+2k}{6}\right)\left[1-(k+3).\left(\frac{1}{n}\right)\right]\right\}.\left(\frac{1}{n}\right)^4\right\}+3+C_4$$

$$14=\left\{\lim_{n\to\infty}\left[\frac{120}{n^4}\sum_{k=1}^{n-3}k^3+\frac{360}{n^4}\sum_{k=1}^{n-3}k^2+\frac{2}{n^4}\sum_{k=1}^{n-3}k-\frac{120}{n^5}\sum_{k=1}^{n-3}k^4-\frac{720}{n^5}\sum_{k=1}^{n-3}k^3-\frac{1{,}320}{n^5}\sum_{k=1}^{n-3}k^2-\frac{720}{n^5}\sum_{k=1}^{n-3}k\right]\right\}+3+C_4$$

NOTA: Como $n\to\infty$, el límite superior de la Sumatoria ***n-3*** puede ser remplazado por ***n***, ya que la diferencia finita de 3 es un valor insignificante, por tanto:

$$\rightarrow 14=\left\{\lim_{n\to\infty}\left[\frac{120}{n^4}\sum_{k=1}^{n-3}k^3+\frac{360}{n^4}\sum_{k=1}^{n-3}k^2+\frac{2}{n^4}\sum_{k=1}^{n-3}k-\frac{120}{n^5}\sum_{k=1}^{n-3}k^4-\frac{720}{n^5}\sum_{k=1}^{n-3}k^3-\frac{1{,}320}{n^5}\sum_{k=1}^{n-3}k^2-\frac{720}{n^5}\sum_{k=1}^{n-3}k\right]\right\}+3+C_4$$

$$14=\left\{\lim_{n\to\infty}\left[\frac{120}{n^4}\sum_{k=1}^{n}k^3+\frac{360}{n^4}\sum_{k=1}^{n}k^2+\frac{2}{n^4}\sum_{k=1}^{n}k-\frac{120}{n^5}\sum_{k=1}^{n}k^4-\frac{720}{n^5}\sum_{k=1}^{n}k^3-\frac{1{,}320}{n^5}\sum_{k=1}^{n}k^2-\frac{720}{n^5}\sum_{k=1}^{n}k\right]\right\}+3+C_4$$

$$14=\left\{\lim_{n\to\infty}\left[\left(\frac{120}{n^4}\right)\left(\frac{n^4}{4}+\cdots\right)+\left(\frac{360}{n^4}\right)\left(\frac{n^3}{3}+\cdots\right)+\left(\frac{2}{n^4}\right)\left(\frac{n^2}{2}+\cdots\right)-\left(\frac{120}{n^5}\right)\left(\frac{n^5}{5}+\cdots\right)-\left(\frac{720}{n^5}\right)\left(\frac{n^4}{4}+\cdots\right)\right.\right.$$
$$\left.\left.-\left(\frac{1{,}320}{n^5}\right)\left(\frac{n^3}{3}+\cdots\right)-\left(\frac{720}{n^5}\right)\left(\frac{n^2}{2}+\cdots\right)\right]\right\}+3+C_4$$

$$11=\left\{\lim_{n\to\infty}\left[30+\left(\frac{120}{n}\right)+\left(\frac{1}{n^2}\right)-24-\left(\frac{180}{n}\right)-\left(\frac{440}{n^2}\right)-\left(\frac{360}{n^3}\right)\right]\right\}+C_4$$

$$11=\left\{\left[\lim_{n\to\infty}30+\lim_{n\to\infty}\left(\frac{120}{n}\right)+\lim_{n\to\infty}\left(\frac{1}{n^2}\right)-\lim_{n\to\infty}24-\lim_{n\to\infty}\left(\frac{180}{n}\right)-\lim_{n\to\infty}\left(\frac{440}{n^2}\right)-\lim_{n\to\infty}\left(\frac{360}{n^3}\right)\right]\right\}+C_4$$

$$11=30+0+0-24-0-0-0+C_4$$

$$11=6+C_4$$

$$\therefore C_4=5\ \blacksquare$$

COMPROBACIÓN

La ecuación ***(m4)*** (**Página 58**) es:

$$\rightarrow f(x_0) \cong \left\{\left\{\sum_{k=1}^{N-3}\left(\frac{k^3+3k^2+2k}{6}\right).f\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\right\}+\frac{1}{6}C_1\,(x_0)^3+\frac{1}{2}C_2(x_0)^2+C_3(x_0)+C_4\cdots\cdots(m4)$$

$$\rightarrow \hat{y} = \left\{\left\{\sum_{k=1}^{100{,}000-3} 720\left(\frac{k^3+3k^2+2k}{6}\right)\left[x_0-\left(\frac{x_0}{100{,}000}\right).(k+3)\right]\right\}.\left(\frac{x_0}{100{,}000}\right)^4\right\}+\frac{1}{6}C_1\,(x_0)^3+\frac{1}{2}C_2(x_0)^2+C_3(x_0)+C_4$$

$$\hat{y} = \left\{\left\{\sum_{k=1}^{99{,}997} 720\left(\frac{k^3+3k^2+2k}{6}\right)\left[x_0-\left(\frac{x_0}{100{,}000}\right).(k+3)\right]\right\}.\left(\frac{x_0}{100{,}000}\right)^4\right\}+\frac{1}{6}C_1\,(x_0)^3+\frac{1}{2}C_2(x_0)^2+C_3(x_0)+C_4\cdots\cdots(m4\omega)$$

Esta representación [en este caso ***(m4ω)***] constituye una **Integral Simbólica**, la cual podemos definir como:

"La integral indefinida de f(x) evaluada numericamente en un punto x_0 por medio de una suma finita de N términos, donde N juega el papel del infinito, produciendo el valor de la antiderivada en ese punto para cualquier grado de precisión deseada"

El valor de las Constantes previamente calculadas son:

$\rightarrow C_1 = 0$

$\rightarrow C_2 = 6.0036$

$\rightarrow C_3 = -0.0018$

$\rightarrow C_4 = 5.0006$

Necesitamos redondear las Constantes, por tanto:

$\rightarrow C_1 = 0$

$\rightarrow C_2 = 6$

$\rightarrow C_3 = 0$

$\rightarrow C_4 = 5$

Por tanto, la ecuación ***(m4ω)*** se convierte en:

$$\rightarrow \hat{y} = \left\{\left\{\sum_{k=1}^{99{,}997} 720\left(\frac{k^3+3k^2+2k}{6}\right)\left[x_0-\left(\frac{x_0}{100{,}000}\right).(k+3)\right]\right\}.\left(\frac{x_0}{100{,}000}\right)^4\right\}+\frac{1}{6}C_1\,(x_0)^3+\frac{1}{2}C_2(x_0)^2+C_3(x_0)+C_4\cdots\cdots(m4\omega)$$

$$\hat{y} = \left\{\left\{\sum_{k=1}^{99{,}997} 720\left(\frac{k^3+3k^2+2k}{6}\right)\left[x_0-\left(\frac{x_0}{100{,}000}\right).(k+3)\right]\right\}.\left(\frac{x_0}{100{,}000}\right)^4\right\}+\frac{1}{6}(0)\,(x_0)^3+\frac{1}{2}(6)(x_0)^2+(0)(x_0)+5\cdots\cdots(m4\omega)$$

$$\rightarrow \hat{y} = \left\{\left\{\sum_{k=1}^{99{,}997} 120(k^3+3k^2+2k)\left[x_0-\left(\frac{x_0}{100{,}000}\right).(k+3)\right]\right\}.\left(\frac{x_0}{100{,}000}\right)^4\right\}+3(x_0)^2+5\cdots\cdots(m4\omega)$$

Para crear una tabulación en la Hoja de Cálculo, necesitamos abrir una quinta Hoja de Cálculo y nombrarla ***m=4 Graph***, y luego copiar todos los datos de la Hoja de Cálculo ***M=4.***

Por lo que:

1) Escribimos la ecuación ***(m4ω)*** en la **celda A1**:

2) Vamos a comenzar en la **celda A10**, ingresamos los siguientes datos:

	A	B	C	D
1	$\hat{y}=\left\{\left\{\sum_{k=1}^{99,997} 120(k^3+3k^2+2k)\left[x_0-\left(\frac{x_0}{100,000}\right).(k+3)\right]\right\}.\left(\frac{x_0}{100,000}\right)^4\right\}+3(x_0)^2+5\cdots\cdots(m4\omega)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0-\left(\frac{x_0}{N}\right)(k+3)\right]$	$120(k^3+3k^2+2k)\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{120(k^3+3k^2+2k)\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right].\left(\frac{x_0}{N}\right)^4\right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000
100021			SUMA	5.9994
100022				
100023			f(x_0)	14
100024		(-)	SUMATORIA	5.9994
100025		(-)	(1/6)C1(calculada previamente)	0
100026		(-)	(1/2)C2(calculada previamente)	3.0018
100027		(-)	C3(calculada previamente)	-0.0018
100028		(=)	C4	5.0006

3) Borramos el contenido de la **celda A13** y la **celda B13** y reemplazamos los contenidos (desde la **celda C100021**):

	A	B	C	D
1	$\hat{y}=\left\{\left\{\sum_{k=1}^{99,997} 120(k^3+3k^2+2k)\left[x_0-\left(\frac{x_0}{100,000}\right).(k+3)\right]\right\}.\left(\frac{x_0}{100,000}\right)^4\right\}+3(x_0)^2+5\cdots\cdots(m4\omega)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13				
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0-\left(\frac{x_0}{N}\right)(k+3)\right]$	$120(k^3+3k^2+2k)\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{120(k^3+3k^2+2k)\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right].\left(\frac{x_0}{N}\right)^4\right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000
100021			SUMATORIA	=SUM(D21:D100017)
100022		(+)	$3(x_0)^2$	=3*(B10)^2
100023		(+)	5	=5
100024		(=)	$f(x_0)$	=D100021+D100022+D100023
100025				
100026				
100027				

De acuerdo a:

$$\hat{y}=\left\{\left\{\sum_{k=1}^{99,997} 120(k^3+3k^2+2k)\left[x_0-\left(\frac{x_0}{100,000}\right).(k+3)\right]\right\}.\left(\frac{x_0}{100,000}\right)^4\right\}+3(x_0)^2+5\cdots\cdots(m4\omega)$$

Por ende, al introducir valores para *x*, tenemos:

	A	B	C	D
10	x_0=	**1**		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13				
100021			**SUMATORIA**	**5.9994**
100022		**(+)**	**$3(x_0)^2$**	3
100023		**(+)**	**5**	5
100024		**(=)**	**$f(x_0)$**	**13.99940**
100025				

Si:

	A	B	C	D
10	x_0=	**2**		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13				
100021			**SUMATORIA**	**191.9808**
100022		**(+)**	**$3(x_0)^2$**	12
100023		**(+)**	**5**	5
100024		**(=)**	**$f(x_0)$**	**208.98080**
100025				

Si:

	A	B	C	D
10	x_0=	**3**		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13				
100021			**SUMATORIA**	**1,457.8542**
100022		**(+)**	**$3(x_0)^2$**	27
100023		**(+)**	**5**	5
100024		**(=)**	**$f(x_0)$**	**1,489.85421**
100025				

La solución real a:

$$\frac{d^4}{dx^4} = 720x$$

Para la condición inicial (x,y,y',y'',y''')=(1,14; 36,126,360)

Es:

$$\rightarrow y = 6x^5 + 3x^2 + 5$$

Si los valores de esta función explicita son tabulados y comparados con los obtenidos por medio de la Hoja de Cálculo, se obtiene lo siguiente:

x	$y = 6x^5 + 3x^2 + 5$	$\hat{y}$	$Difference$
-3	-1,426	-1,425.8542	0.14579
-2	-175	-174.98080	0.01920
-1	2	2.00060	0.00060
0	5	5.0000	0.00000
1	14	13.99940	-0.00060
2	209	208.98080	-0.01920
3	1,490	1,489.85421	-0.14579
4	6,197	6,196.38562	-0.61438
5	18,830	18,828.1250	-1.87493

$$\hat{y} = \left\{ \left\{ \sum_{k=1}^{99,997} 120(k^3 + 3k^2 + 2k) \left[x_0 - \left(\frac{x_0}{100,000} \right) . (k+3) \right] \right\} . \left(\frac{x_0}{100,000} \right)^4 \right\} + 3(x_0)^2 + 5 \cdots\cdots (m4\omega)$$

Como puedes ver, estas ***pequeñas diferencias*** indican que ***N=100,000*** **es una buena aproximación al infinito (∞)**, pero, si una ***N=1,000,000 o más grande fueran elegidas***, esas ***diferencias tenderían a cero***.

Por tanto, este método es una manera de **calcular antiderivadas** numéricamente usando Hoja de Cálculo.

INTEGRALES DEFINIDAS

RESOLVIENDO INTEGRALES DEFINIDAS CON HOJA DE CÁLCULO

Para encontrar una ***Integral Definida*** a partir de una **Integral Indefinida**, se usa la siguiente definición:

	$\left\{\lim_{n\to\infty}\left\{\left\{\sum_{k=1}^{n}\left[f\left(b-\frac{bk}{n}\right)\right]\right\}\times\left(\frac{b}{n}\right)\right\}\right\}$
(-)	$\left\{\lim_{n\to\infty}\left\{\left\{\sum_{k=1}^{n}\left[f\left(a-\frac{ak}{n}\right)\right]\right\}\times\left(\frac{a}{n}\right)\right\}\right\}$
(=)	$\int_a^b f(x)\,dx$

En el caso de una Hoja de Cálculo, una aproximación a la Integral Definida será con ***un número muy grande N que tienda hacia el infinito,*** el cual puede ser ***N=100,000 o más grande***, así la anterior definición se convierte en lo siguiente:

	$\left\{\left\{\sum_{k=1}^{N}\left[f\left(b-\frac{bk}{N}\right)\right]\right\}\times\left(\frac{b}{N}\right)\right\}$	
$(-)$	$\left\{\left\{\sum_{k=1}^{N}\left[f\left(a-\frac{ak}{N}\right)\right]\right\}\times\left(\frac{a}{N}\right)\right\}$	$\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\psi)$
$(\cong)$	$\int_a^b f(x)\,dx$	

Note que la definición de arriba que implica una resta, corresponde a la ***Primera Integral Indefinida*** [el cual se obtiene al sustituir **δ=-1** en la **Definición (1.1)**].

EJEMPLO:

Encuentre:

$$0.398942\int_{a=-1}^{b=1} e^{-\frac{1}{2}x^2}\,dx$$

Usando una tendencia hacia el infinito con ***N=100,000***

Sol…

Aplicando la ***Definición (ψ)***:

	$\left\{\left\{\sum_{k=1}^{N}\left[f\left(b-\frac{bk}{N}\right)\right]\right\}\times\left(\frac{b}{N}\right)\right\}$
$(-)$	$\left\{\left\{\sum_{k=1}^{N}\left[f\left(a-\frac{ak}{N}\right)\right]\right\}\times\left(\frac{a}{N}\right)\right\}$
$(\cong)$	$\int_{a}^{b} f(x)\,dx$

$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(\psi)$

Por tanto:

$$0.398942\int_{a=-1}^{b=1} e^{-\frac{1}{2}x^2}\,dx$$

Es:

	$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$
$(-)$	$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$
$(\cong)$	$0.398942\int_{a=-1}^{b=1} e^{-\frac{1}{2}x^2}\,dx$

NOTA: Haremos esto como cuando la Ecuación (M1) se usó para calcular Antiderivadas.

1) Ingresamos la Integral Definida en la **celda A1**:
2) Lo siguiente será escrito en la **celda A10** y **la celda E10**:
3) Escribimos en la **celda A20**:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	=(B10)/(B11)			(a/N)=	=(F10)/(F11)		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21								
22								
23								

De acuerdo a:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$$

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$$

4) Escribimos en la **celda A21** y la **celda E21**:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21								
22								
23								

=SEQUENCE(100000,1,1,1)

Esto significa que habrá **100,000 Filas**
En **1 Columna**
Comenzado **desde 1**
Incrementándose en 1 (desde 1 hasta llegar a N=100,000)

5) Por tanto, se verá como sigue:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1				1			
22	2				2			
23	3				3			
24	4				4			
25	5				5			
100019	9,9999				9,9999			
100020	100,000				100,000			

6) Ahora en la **celda B21** y la **celda E21,** escribimos lo siguiente (después de autorellenar hasta ***k=100,000***):

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1				1			
22	2				2			
23	3				3			
24	4				4			
25	5				5			
100019	9,9999				9,9999			
100020	100,000				100,000			

=(B10)-(B12)*(A21)

Representa en Rojo:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$$

=(F10)-(F12)*(E21)

Representa en Rojo:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$$

7) Se verá como sigue:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999			1	-0.99999		
22	2	0.99998			2	-0.99998		
23	3	0.99997			3	-0.99997		
24	4	0.99996			4	-0.99996		
25	5	0.99995			5	-0.99995		
100019	9,9999	0.00001			9,9999	-0.00001		
100020	100,000	0.00000			100,000	0.00000		

8) En la **celda C21** y la **celda G21**, escribimos lo siguiente (después de autorellenar hasta ***k=100,000***):

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999			1	-0.99999		
22	2	0.99998			2	-0.99998		
23	3	0.99997			3	-0.99997		
24	4	0.99996			4	-0.99996		
25	5	0.99995			5	-0.99995		
100019	9,9999	0.00001			9,9999	-0.00001		
100020	100,000	0.00000			100,000	0.00000		

=exp(-(1/2)*(B21)^2)

Representa en Rojo:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$$

=exp(-(1/2)*(F21)^2)

Representa en Rojo:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$$

9) Se verá como sigue:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654		1	-0.99999	0.60654	
22	2	0.99998	0.60654		2	-0.99998	0.60654	
23	3	0.99997	0.60655		3	-0.99997	0.60655	
24	4	0.99996	0.60655		4	-0.99996	0.60655	
25	5	0.99995	0.60656		5	-0.99995	0.60656	
100019	9,9999	0.00001	1.00000		9,9999	-0.00001	1.00000	
100020	100,000	0.00000	1.00000		100,000	0.00000	1.00000	

10) En la **celda D21** y la **celda H21**, escribimos lo siguiente (después de autorellenar hasta ***k=100,000***)::

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654		1	-0.99999	0.60654	
22	2	0.99998	0.60654		2	-0.99998	0.60654	
23	3	0.99997	0.60655		3	-0.99997	0.60655	
24	4	0.99996	0.60655		4	-0.99996	0.60655	
25	5	0.99995	0.60656		5	-0.99995	0.60656	
100019	9,9999	0.00001	1.00000		9,9999	-0.00001	1.00000	
100020	100,000	0.00000	1.00000		100,000	0.00000	1.00000	

=(C21)*(B12)

Representa en Rojo:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$$

Equivalente a:

$$0.398942\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)\right\}$$

=(G21)*(F12)

Representa en Rojo:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$$

Equivalente a:

$$0.398942\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)\right\}$$

11) Se verá como sigue:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021								

12) En la **celda C100021** y **G100021** escribimos **"SUM B"** y **"SUM A"**, respectivamente:

13) En la **celda D100021** y **celda H100021**, escribimos:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021			**SUM B**				**SUM A**	

=SUM(D21:D100020)

Representa en Rojo:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$$

Equivalente a:

$$0.398942\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)\right\}$$

=SUM(H21:H100020)

Representa en Rojo:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$$

Equivalente a:

$$0.398942\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)\right\}$$

14) Se verá como sigue:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021			**SUM B**	**0.85563**			**SUM A**	**-0.85563**
100022								
100023								
100024								

15) Ahora, en la **celda C100023** y la **celda G100023**, escribimos (debido al limitado espacio) las palabras **0.3XSUM B** y **0.3XSUM A**, respectivamente.

16) Mientras que en la **celda D100023** y la **celda H100023** escribimos lo siguiente:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021			**SUM B**	**0.85563**			**SUM A**	**-0.85563**
100022								
100023			**0.3XSUM B**				**0.3XSUM A**	
100024								

=0.398942*(D100021)

Representa en Rojo:

$$0.398942\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)\right\}$$

=0.398942*(H100021)

Representa en Rojo:

$$0.398942\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)\right\}$$

17) Se verá como sigue:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021			**SUM B**	**0.85563**			**SUM A**	**-0.85563**
100022								
100023			**0.3XSUM B**	**0.34135**			**0.3XSUM A**	**-0.34135**
100024								

18) Ahora en la **celda C100024**, escribimos la palabra "**DIFERENCIA**".

19) Mientras que en la **celda D100024**, escribimos:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021			**SUM B**	**0.85563**			**SUM A**	**-0.85563**
100022								
100023			**0.3XSUM B**	**0.34135**			**0.3XSUM A**	**-0.34135**
100024			**DIFERENCIA**					

=D100023-H100023

Representa en Rojo:

$$\mathbf{0.398942}\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)\right\}-\mathbf{0.398942}\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)\right\}$$

De acuerdo a:

	$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$
(−)	$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$
(≅)	$0.398942\int_{a=-1}^{b=1} e^{-\frac{1}{2}x^2}dx$

20) **Finalmente**, se verá como sigue:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021			**SUM B**	**0.85563**			**SUM A**	**-0.85563**
100022								
100023			**0.3XSUM B**	**0.34135**			**0.3XSUM A**	**-0.34135**
100024			**DIFERENCIA**	**0.68269**				

Podemos verificar que:

$$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx \cong 0.682689492$$

¡Por tanto, el resultado en la Hoja de Cálculo de 0.68269 es una buena aproximación dada una N=100,000!

REFERENCIAS BIBLIOGRÁFICAS

- Para el link al libro "***On Indefinite Integral***" (e-book en archive.org):

- Para el link al Manual para calcular ***Antiderivadas*** e ***Integrales Definidas*** en Español (e-book en archive.org):

- Para el link a este Manual en idioma Inglés (e-book en academia.edu):

- Para ver la demostración completa de cómo surge la ***Primera Definición de Integral Indefinida*** (vídeo en Youtube):

Este manual presenta un método numérico para calcular ***Antiderivadas*** e ***Integrales Definidas*** utilizando software de hoja de cálculo, basado en el marco teórico desarrollado en "***On Indefinite Integral***" (**ISBN 978-607-29-3804-5**). El método interpreta la ***Integral Indefinida*** como ***una derivada de orden negativo*** y emplea Sumas Finitas con valores grandes de ***N*** como aproximación práctica a la definición de *límite*, lo que permite el cálculo de *antiderivadas* sin necesidad de software matemático especializado.

Mediante cuatro pasos de integración sucesiva aplicados a una ecuación diferencial de cuarto orden, el manual demuestra cómo se determinan numéricamente las constantes de integración utilizando condiciones iniciales y ***cómo se recupera la Función Original paso a paso***. Una sección aparte extiende el método a *integrales definidas*, ilustrado con la función gaussiana. Todos los procedimientos se implementan exclusivamente en la hoja de cálculo ***WPS Office***, utilizando funciones estándar disponibles en cualquier aplicación de hoja de cálculo moderna.

Este trabajo pretende ser una herramienta pedagógica para estudiantes de pregrado de matemáticas, ingeniería y campos afines, ofreciendo un enfoque algebraico y aritmético del cálculo integral que reduce su complejidad operativa a la vez que preserva el rigor matemático.